Quantum Mechanics
Tutorial Problems

Engineering Physics

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Q1. Find the wavelength of (a) a 46-g golf ball with a velocity 30 m/s and, (b) an electron with a velocity of 10^7 m/s. **[Ans. (a) 4.8 x 10^{-34} m, (b) 7.28 x 10^{-11} m]**

Q2. An electron has a de Broglie wavelength of 2×10^{-12} m. Find its kinetic energy, phase velocity and group velocity. **[Ans. K.E. = 292.9 keV, v_p = 1.3 c, v_g = 0.772 c]**

Q3. Two harmonics waves represented by $\xi_1 = 3\cos(7t - 10x)$ m and $\xi_2 = 3\cos(3t - 8x)$ m are superposed to form a wave group. Find the group velocity. **[Ans. 2 m/s]**

Q4. The radius of the hydrogen atom is 5.3×10^{-11} m. Use the uncertainty principle to estimate the minimum energy of an electron in this atom. **[Ans. 0.85 eV]**

- **The lifetime of a nucleus in an excited state is 10^{-12} s. Calculate the probable uncertainty in the energy and frequency of γ-ray photon emitted by it.**

The energy-time uncertainty relation is $\Delta E\,\Delta t \approx \frac{\hbar}{2}$

$$\therefore \Delta E \approx \frac{\hbar}{2\,\Delta t} = \frac{1.054\times10^{-34}\,J.s}{2\times10^{-12}\,s} = 0.527\times10^{-22}\,J$$

The uncertainty in frequency is

$$\Delta\nu = \frac{\Delta E}{h} = \frac{0.527\times10^{-22}\,J}{6.626\times10^{-34}\,J.s} = 0.795\times10^{11}\,Hz$$

- **The average lifetime of an excited atomic state is 10^{-8}s. If the wavelength of the spectral line associated with the transition from this state to the ground state is 6000 Å, estimate the width of this line.**

Since $E = h\nu = \frac{hc}{\lambda}$, we have $\Delta E = -\frac{hc}{\lambda^2}\,\Delta\lambda$

According to the uncertainty principle, $\Delta E\,\Delta t = \frac{\hbar}{2}$

$$\therefore -\frac{hc}{\lambda^2}\,\Delta\lambda\,\Delta t = \frac{\hbar}{2}$$

$$\therefore |\Delta\lambda| = \frac{\lambda^2}{4\pi c\,\Delta t} = \frac{(6\times10^{-7})^2}{4\times3.14\times3\times10^{8}\times10^{-8}} = 0.955\times10^{-14}\,m$$

➢ The Davisson – Germer experiment: An experiment that confirms the existence of de Broglie waves

Measured by XRD

n = 1, θ = 65^0 (highest intensity observed with a 54 V) and d = 0.091 nm (spacing of crystalline planes of nickel)

The Bragg equation for maxima in the diffraction pattern

$$n\lambda = 2d \sin\theta = 2(0.091\ \text{nm})(\sin 65^0) = 0.165\ \text{nm}$$

Now we use de Broglie's formula to find expected wavelength of the electrons i.e. $\lambda = \frac{h}{\gamma m v}$

Kinetic energy of electron KE = eV = 54 eV

since KE < 0.51 MeV (rest energy of electron). So we can let $\gamma = 1$

We also know that $K = \frac{1}{2} m v^2 = \frac{p^2}{2m}$ Gives $p = \sqrt{2mKE}$

$$\lambda = \frac{h}{\sqrt{2mKE}} = \frac{6.63\times10^{-34}\ J.s}{\sqrt{2(9.1\times10^{-31}\ kg)(54eV)\left(1.6\times10^{-19}\frac{J}{eV}\right)}} = 0.166\ \text{nm}$$

Which agrees well with the observed wavelength of 0.165 nm. The Davisson - Germer experiments thus directly verifies de Broglie hypothesis of the wave nature of moving bodies.

Q5. In the Davisson-Germer Experiment, if the electron beam was accelerated by 100 volts, at which scattering angle would they have found a peak in the intensity? The spacing between two crystalline planes in Nickel is 0.091 nm. **[Ans. 95°]**

Q6. A beam of neutrons that emerges from a nuclear reactor contains neutrons with a variety of energies. To obtain neutrons with an energy of 0.050 eV, the beam is passed through a crystal whose atomic planes are 0.20 nm apart. At what angles relative to the original beam will the desired neutrons be diffracted? **[142.64°]**

Q7. Normalize the following wave fuctions:

(a) $\Psi(y) = A \exp.(-y^2)$, for $0<y<\infty$

(b) $\Psi(x) = A \sin^3(\pi x/a)$, for $0<x<a$